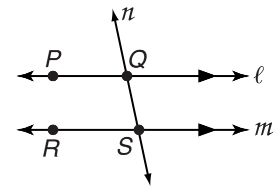
NAME ____ DATE _____________ PERIOD _____

3-1 Study Guide and Intervention

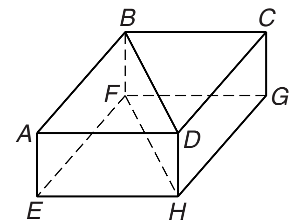
Parallel Lines and Transversals

Relationships Between Lines and Planes

When two lines lie in the same plane and do not intersect, they are **parallel**. Lines that do not intersect and are not coplanar are **skew lines**. In the figure, ℓ is parallel to m, or $\ell \parallel m$. You can also write $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$. Similarly, if two planes do not intersect, they are **parallel planes**.



Example



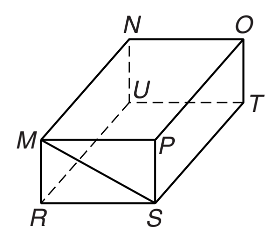
a. Name all planes that are parallel to plane *ABD*.
plane *EFH*

b. Name all segments that are parallel to $\overline{CG}$.
$\overline{BF}$, $\overline{DH}$, and $\overline{AE}$

c. Name all segments that are skew to $\overline{EH}$.
$\overline{BF}$, $\overline{CG}$, $\overline{BD}$, $\overline{CD}$, and $\overline{AB}$

Exercises

For Exercises 1–3, refer to the figure at the right.

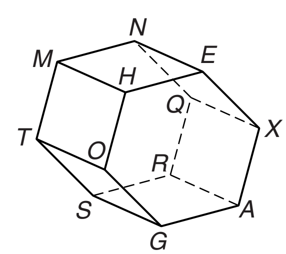


1. Name all planes that intersect plane *OPT*.

2. Name all segments that are parallel to $\overline{NU}$.

3. Name all segments that intersect $\overline{MP}$.

For Exercises 4–7, refer to the figure at the right.



4. Name all segments parallel to $\overline{QX}$.

5. Name all planes that intersect plane *MHE*.

6. Name all segments parallel to $\overline{QR}$.

7. Name all segments skew to $\overline{AG}$.

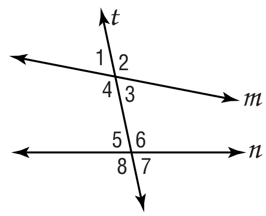
NAME ______________________________________ DATE __________ PERIOD _____

3-1 Study Guide and Intervention *(continued)*

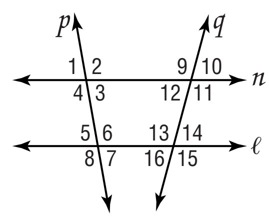
Parallel Lines and Transversals

Angle Relationships A line that intersects two or more other lines in a plane is called a **transversal**. In the figure below, t is a transversal. Two lines and a transversal form eight angles. Some pairs of the angles have special names. The following chart lists the pairs of angles and their names.

Angle Pairs	Name
$\angle 3$, $\angle 4$, $\angle 5$, and $\angle 6$	interior angles
$\angle 3$ and $\angle 5$; $\angle 4$ and $\angle 6$	alternate interior angles
$\angle 3$ and $\angle 6$; $\angle 4$ and $\angle 5$	consecutive interior angles
$\angle 1$, $\angle 2$, $\angle 7$, and $\angle 8$	exterior angles
$\angle 1$ and $\angle 7$; $\angle 2$ and $\angle 8$	alternate exterior angles
$\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 3$ and $\angle 7$; $\angle 4$ and $\angle 8$	corresponding angles



Example **Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.**



a. **$\angle 10$ and $\angle 16$**
alternate exterior angles

b. **$\angle 4$ and $\angle 12$**
corresponding angles

c. **$\angle 12$ and $\angle 13$**
consecutive interior angles

d. **$\angle 3$ and $\angle 9$**
alternate interior angles

Exercises

Use the figure in the Example for Exercises 1–12.

Name the transversal that forms each pair of angles.

1. $\angle 9$ and $\angle 13$

2. $\angle 5$ and $\angle 14$

3. $\angle 4$ and $\angle 6$

Identify each pair of angles as *alternate interior*, *alternate exterior*, *corresponding*, or *consecutive interior* angles.

4. $\angle 1$ and $\angle 5$

5. $\angle 6$ and $\angle 14$

6. $\angle 2$ and $\angle 8$

7. $\angle 3$ and $\angle 11$

8. $\angle 12$ and $\angle 3$

9. $\angle 4$ and $\angle 6$

10. $\angle 6$ and $\angle 16$

11. $\angle 11$ and $\angle 14$

12. $\angle 10$ and $\angle 16$

3-2 Study Guide and Intervention

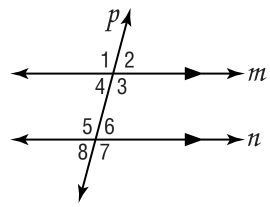
Angles and Parallel Lines

Parallel Lines and Angle Pairs When two parallel lines are cut by a transversal, the following pairs of angles are congruent.

- corresponding angles
- alternate interior angles
- alternate exterior angles

Also, consecutive interior angles are supplementary.

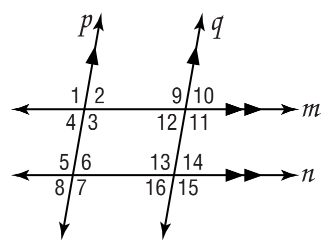
Example **In the figure, $m\angle 2 = 75$. Find the measures of the remaining angles.**



$m\angle 1 = 105$ $\angle 1$ and $\angle 2$ form a linear pair.
$m\angle 3 = 105$ $\angle 3$ and $\angle 2$ form a linear pair.
$m\angle 4 = 75$ $\angle 4$ and $\angle 2$ are vertical angles.
$m\angle 5 = 105$ $\angle 5$ and $\angle 3$ are alternate interior angles.
$m\angle 6 = 75$ $\angle 6$ and $\angle 2$ are corresponding angles.
$m\angle 7 = 105$ $\angle 7$ and $\angle 3$ are corresponding angles.
$m\angle 8 = 75$ $\angle 8$ and $\angle 6$ are vertical angles.

Exercises

In the figure, $m\angle 3 = 102$. Find the measure of each angle.

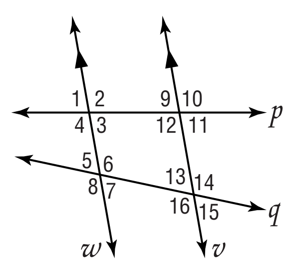


1. $\angle 5$ **2.** $\angle 6$

3. $\angle 11$ **4.** $\angle 7$

5. $\angle 15$ **6.** $\angle 14$

In the figure, $m\angle 9 = 80$ and $m\angle 5 = 68$. Find the measure of each angle.



7. $\angle 12$ **8.** $\angle 1$

9. $\angle 4$ **10.** $\angle 3$

11. $\angle 7$ **12.** $\angle 16$

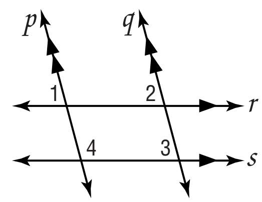
Lesson 3-2

3-2 Study Guide and Intervention *(continued)*

Angles and Parallel Lines

Algebra and Angle Measures Algebra can be used to find unknown values in angles formed by a transversal and parallel lines.

Example **If $m\angle 1 = 3x + 15$, $m\angle 2 = 4x - 5$, $m\angle 3 = 5y$, and $m\angle 4 = 6z + 3$, find x and y.**



$p \parallel q$, so $m\angle 1 = m\angle 2$ because they are corresponding angles.

$$\begin{aligned} 3x + 15 &= 4x - 5 \\ 3x + 15 - 3x &= 4x - 5 - 3x \\ 15 &= x - 5 \\ 15 + 5 &= x - 5 + 5 \\ 20 &= x \end{aligned}$$

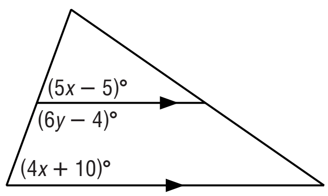
$r \parallel s$, so $m\angle 2 = m\angle 3$ because they are corresponding angles.

$$\begin{aligned} m\angle 2 &= m\angle 3 \\ 75 &= 5y \\ \frac{75}{5} &= \frac{5y}{5} \\ 15 &= y \end{aligned}$$

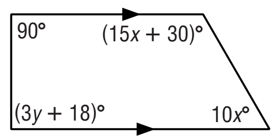
Exercises

Find x and y in each figure.

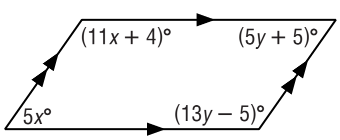
1.



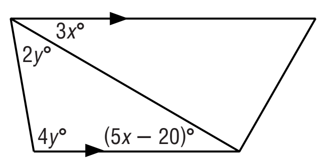
2.



3.

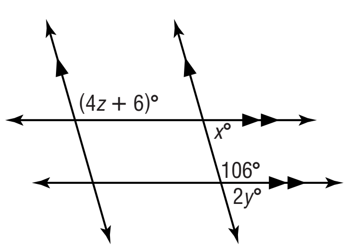


4.

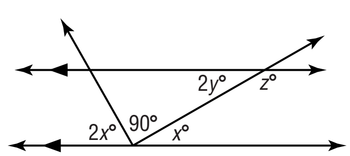


Find x, y, and z in each figure.

5.



6.

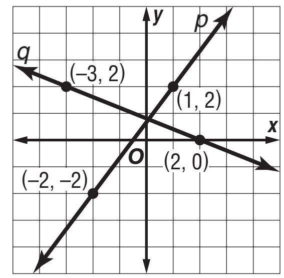


3-3 Study Guide and Intervention

Slopes of Lines

Slope of a Line The slope m of a line containing two points with coordinates (x_1, y_1) and (x_2, y_2) is given by the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$, where $x_1 \neq x_2$.

Example **Find the slope of each line.**



For line p, let (x_1, y_1) be (1, 2) and (x_2, y_2) be (−2, −2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-2 - 2}{-2 - 1} \text{ or } \frac{4}{3}$$

For line q, let (x_1, y_1) be (2, 0) and (x_2, y_2) be (−3, 2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - 0}{-3 - 2} \text{ or } -\frac{2}{5}$$

Exercises

Determine the slope of the line that contains the given points.

1. $J(0, 0)$, $K(-2, 8)$

2. $R(-2, -3)$, $S(3, -5)$

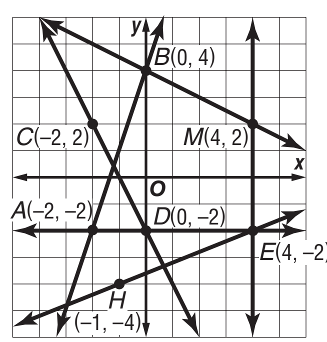
3. $L(1, -2)$, $N(-6, 3)$

4. $P(-1, 2)$, $Q(-9, 6)$

5. $T(1, -2)$, $U(6, -2)$

6. $V(-2, 10)$, $W(-4, -3)$

Find the slope of each line.



7. $\overleftrightarrow{AB}$

8. $\overleftrightarrow{CD}$

9. $\overleftrightarrow{EM}$

10. $\overleftrightarrow{AE}$

11. $\overleftrightarrow{EH}$

12. $\overleftrightarrow{BM}$

3-3 Study Guide and Intervention *(continued)*

Slopes of Lines

Parallel and Perpendicular Lines If you examine the slopes of pairs of parallel lines and the slopes of pairs of perpendicular lines, where neither line in each pair is vertical, you will discover the following properties.

Two lines have the same slope if and only if they are parallel.

Two lines are perpendicular if and only if the product of their slopes is -1.

Example 1 **Find the slope of a line parallel to the line containing $A(-3, 4)$ and $B(2, 5)$.**

Find the slope of $\overleftrightarrow{AB}$. Use $(-3, 4)$ for (x_1, y_1) and use $(2, 5)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 4}{2 - (-3)} \text{ or } \frac{1}{5}$$

The slope of any line parallel to $\overleftrightarrow{AB}$ must be $\frac{1}{5}$.

Example 2 **Find the slope of a line perpendicular to $\overleftrightarrow{PQ}$ for $P(-2, -4)$ and $Q(4, 3)$.**

Find the slope of $\overleftrightarrow{PQ}$. Use $(-2, -4)$ for (x_1, y_1) and use $(4, 3)$ for (x_2, y_2).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - (-4)}{4 - (-2)} \text{ or } \frac{7}{6}$$

Since $\frac{7}{6} \cdot \left(-\frac{6}{7}\right) = -1$, the slope of any line perpendicular to $\overleftrightarrow{PQ}$ must be $-\frac{6}{7}$.

Exercises

Determine whether $\overleftrightarrow{MN}$ and $\overleftrightarrow{RS}$ are *parallel*, *perpendicular*, or *neither*.

1. $M(0, 3)$, $N(2, 4)$, $R(2, 1)$, $S(8, 4)$

2. $M(-1, 3)$, $N(0, 5)$, $R(2, 1)$, $S(6, -1)$

3. $M(-1, 3)$, $N(4, 4)$, $R(3, 1)$, $S(-2, 2)$

4. $M(0, -3)$, $N(-2, -7)$, $R(2, 1)$, $S(0, -3)$

5. $M(-2, 2)$, $N(1, -3)$, $R(-2, 1)$, $S(3, 4)$

6. $M(0, 0)$, $N(2, 4)$, $R(2, 1)$, $S(8, 4)$

Find the slope of $\overleftrightarrow{MN}$ and the slope of any line perpendicular to $\overleftrightarrow{MN}$.

7. $M(2, -4)$, $N(-2, -1)$

8. $M(1, 3)$, $N(-1, 5)$

9. $M(4, -2)$, $N(5, 3)$

10. $M(2, -3)$, $N(-4, 1)$

3-4 Study Guide and Intervention

Equations of Lines

Write Equations of Lines You can write an equation of a line if you are given any of the following:

- the slope and the y-intercept,
- the slope and the coordinates of a point on the line, or
- the coordinates of two points on the line.

If m is the slope of a line, b is its y-intercept, and (x_1, y_1) is a point on the line, then:

- the **slope-intercept form** of the equation is $y = mx + b$,
- the **point-slope form** of the equation is $y - y_1 = m(x - x_1)$.

Example 1 **Write an equation in slope-intercept form of the line with slope −2 and y-intercept 4.**

$y = mx + b$ Slope-intercept form

$y = -2x + 4$ $m = -2$, $b = 4$

The slope-intercept form of the equation of the line is $y = -2x + 4$.

Example 2 **Write an equation in point-slope form of the line with slope $-\frac{3}{4}$ that contains (8, 1).**

$y - y_1 = m(x - x_1)$ Point-slope form

$y - 1 = -\frac{3}{4}(x - 8)$ $m = -\frac{3}{4}$, $(x_1, y_1) = (8, 1)$

The point-slope form of the equation of the line is $y - 1 = -\frac{3}{4}(x - 8)$.

Exercises

Write an equation in slope-intercept form of the line having the given slope and y-intercept.

1. m: 2, y-intercept: -3

2. m: $-\frac{1}{2}$, y-intercept: 4

3. m: $\frac{1}{4}$, y-intercept: 5

4. m: 0, y-intercept: -2

5. m: $-\frac{5}{3}$, y-intercept: $\frac{1}{3}$

6. m: -3, y-intercept: -8

Write an equation in point-slope form of the line having the given slope that contains the given point.

7. $m = \frac{1}{2}$, $(3, -1)$

8. $m = -2$, $(4, -2)$

9. $m = -1$, $(-1, 3)$

10. $m = \frac{1}{4}$, $(-3, -2)$

11. $m = -\frac{5}{2}$, $(0, -3)$

12. $m = 0$, $(-2, 5)$

3-4 Study Guide and Intervention *(continued)*

Equations of Lines

Write Equations to Solve Problems Many real-world situations can be modeled using linear equations.

Example **Donna offers computer services to small companies in her city. She charges \$55 per month for maintaining a web site and \$45 per hour for each service call.**

a. Write an equation to represent the total monthly cost C for maintaining a web site and for h hours of service calls.

For each hour, the cost increases \$45. So the rate of change, or slope, is 45. The y-intercept is located where there are 0 hours, or \$55.

$C = mh + b$

$= 45h + 55$

b. Donna may change her costs to represent them by the equation $C = 25h + 125$, where \$125 is the fixed monthly fee for a web site and the cost per hour is \$25. Compare her new plan to the old one if a company has $5\frac{1}{2}$ hours of service calls. Under which plan would Donna earn more?

First plan

For $5\frac{1}{2}$ hours of service Donna would earn

$C = 45h + 55 = 45\left(5\frac{1}{2}\right) + 55$

$= 247.5 + 55$ or \$302.50

Second Plan

For $5\frac{1}{2}$ hours of service Donna would earn

$C = 25h + 125 = 25(5.5) + 125$

$= 137.5 + 125$ or \$262.50

Donna would earn more with the first plan.

Exercises

For Exercises 1–4, use the following information.

Jerri's current satellite television service charges a flat rate of \$34.95 per month for the basic channels and an additional \$10 per month for each premium channel. A competing satellite television service charges a flat rate of \$39.99 per month for the basic channels and an additional \$8 per month for each premium channel.

1. Write an equation in slope-intercept form that models the total monthly cost for each satellite service, where p is the number of premium channels.

2. If Jerri wants to include three premium channels in her package, which service would be less, her current service or the competing service?

3. A third satellite company charges a flat rate of \$69 for all channels, including the premium channels. If Jerri wants to add a fourth premium channel, which service would be least expensive?

4. Write a description of how the fee for the number of premium channels is reflected in the equation.

Lesson 3-4

NAME ______________________ DATE ____________ PERIOD _____

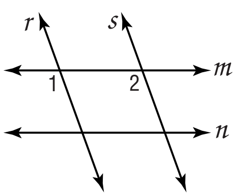
3-5 Study Guide and Intervention

Proving Lines Parallel

Identify Parallel Lines If two lines in a plane are cut by a transversal and certain conditions are met, then the lines must be parallel.

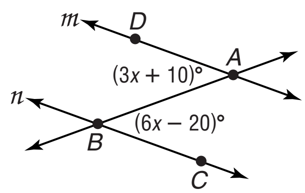
If	then
• corresponding angles are congruent, • alternate exterior angles are congruent, • consecutive interior angles are supplementary, • alternate interior angles are congruent, or • two lines are perpendicular to the same line,	the lines are parallel.

Example 1 **If $m\angle 1 = m\angle 2$, determine which lines, if any, are parallel.**



Since $m\angle 1 = m\angle 2$, then $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 2$ are congruent corresponding angles, so $r \parallel s$.

Example 2 **Find x and $m\angle ABC$ so that $m \parallel n$.**



We can conclude that $m \parallel n$ if alternate interior angles are congruent.

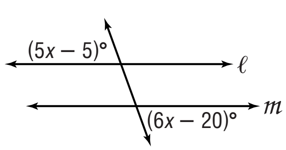
$$\begin{aligned} m\angle BAD &= m\angle ABC \\ 3x + 10 &= 6x - 20 \\ 10 &= 3x - 20 \\ 30 &= 3x \\ 10 &= x \end{aligned}$$

$$\begin{aligned} m\angle ABC &= 6x - 20 \\ &= 6(10) - 20 \text{ or } 40 \end{aligned}$$

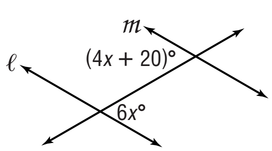
Exercises

Find x so that $\ell \parallel m$.

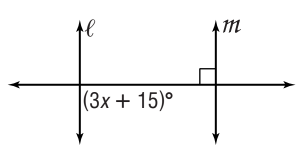
1.



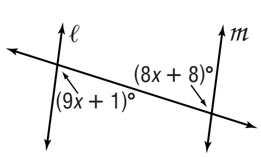
2.



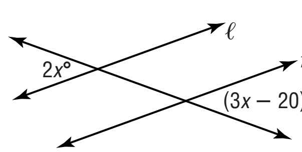
3.



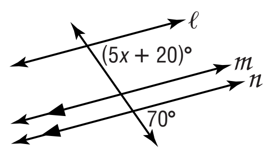
4.



5.



6.



Lesson 3-5