CALCULUS FLASH CARDS

Based on those prepared by Gertrude R. Battaly (http://www.battaly.com/), with her gracious permission.

Instructions for Using the Flash Cards:

1. Cut along the horizontal lines only.
2. Fold along the vertical lines. This will result in flash "cards" with the term on one side and the definition or equivalent expression on the other. You may choose to tape or glue this paper card to a 3×5 card.
3. Use the flash cards at least 10 minutes a day. If you know the definition or formula, put it away for this session. If you don't know it, put it at the back of the stack and do it again.
4. Work alone or with a partner.
5. You may work at school, at home, on the bus to a game, or any place where you can pull the cards out. Every time you use them you will be working towards a good grade on the AP Calculus exam.

$\sin 0$	0
$\sin \frac{\pi}{6}$	$\frac{1}{2}$
$\sin \frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\sin \frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\sin \frac{\pi}{2}$	1

$\sin \pi$	0
$\sin \dfrac{3\pi}{2}$	–1
$\sin 2\pi$	0
$\cos 0$	1
$\cos \dfrac{\pi}{6}$	$\dfrac{\sqrt{3}}{2}$

$\cos \frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\cos \frac{\pi}{3}$	$\frac{1}{2}$
$\cos \frac{\pi}{2}$	0
$\cos \pi$	-1
$\cos \frac{3\pi}{2}$	0

$\cos 2\pi$	1
$\tan\ \theta$ (in terms of sine and/or cosine)	$\dfrac{\sin\theta}{\cos\theta}$
$\cot\ \theta$ (in terms of sine and/or cosine)	$\dfrac{\cos\theta}{\sin\theta}$
csc θ (in terms of sine and/or cosine)	$\dfrac{1}{\sin\theta}$
sec θ (in terms of sine and/or cosine)	$\dfrac{1}{\cos\theta}$

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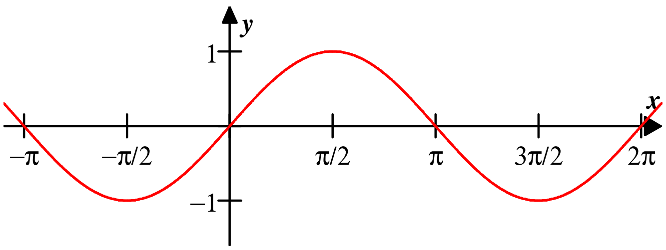
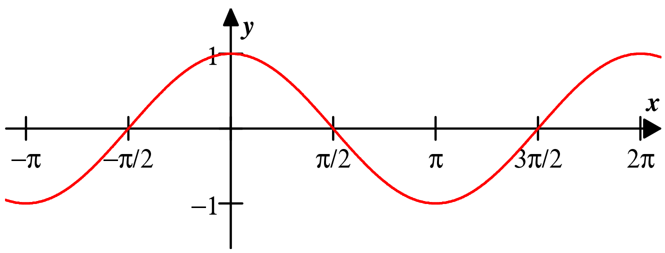
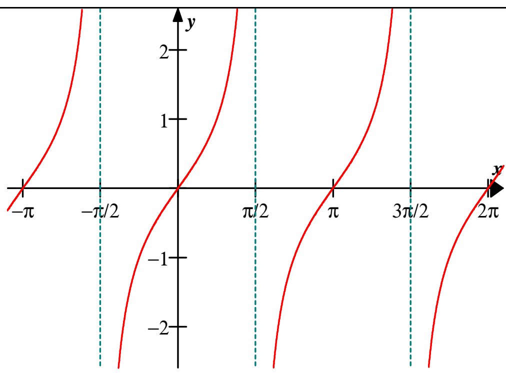
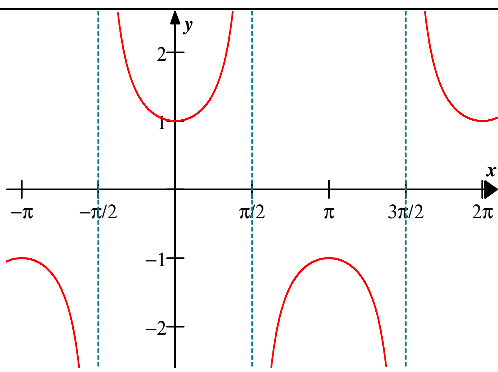
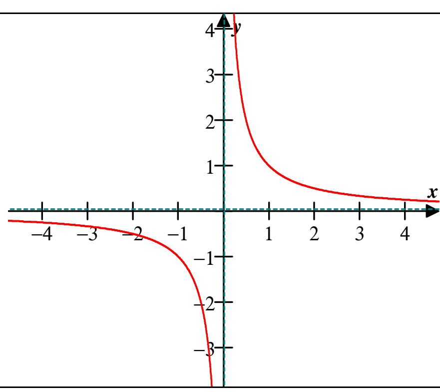
<tr><td>
</td><td>Graph of $y = \sin x$</td></tr>
<tr><td>
</td><td>Graph of $y = \cos x$</td></tr>
<tr><td>
</td><td>Graph of $y = \tan x$</td></tr>
<tr><td>
</td><td>Graph of $y = \sec x$</td></tr>
<tr><td>
</td><td>Graph of $y = \dfrac{1}{x}$</td></tr>
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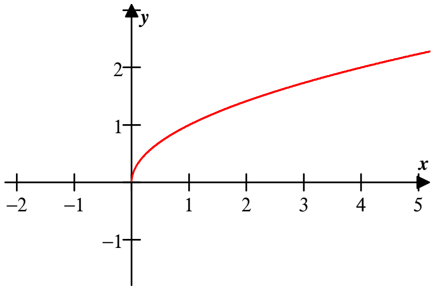
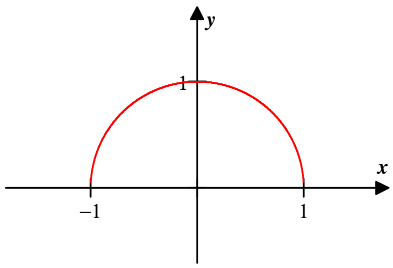
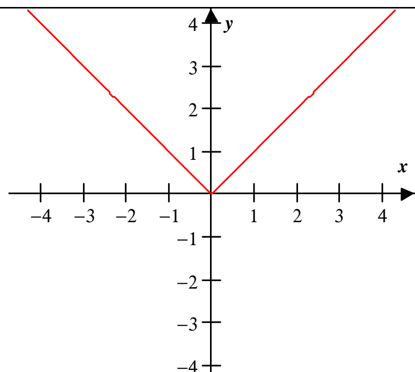


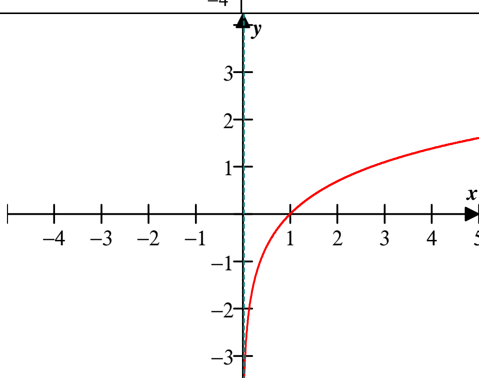
Graph of $y = \sqrt{x}$



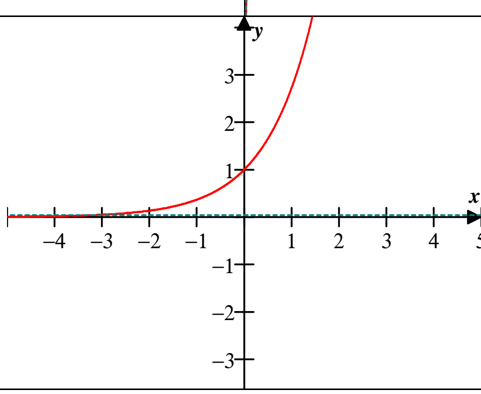
Graph of $y = \sqrt{1 - x^2}$



Graph of $y = |x|$



Graph of $y = \ln x$



Graph of $y = e^x$

Definition: An even function is…	…symmetric with respect to the y-axis, like $y = x^2$, $y = \cos x$, or $y = \lvert x \rvert$. $f(-x) = f(x)$
Definition: An odd function is…	…symmetric with respect to the origin, like $y = x^3$, $y = \sin x$, or $y = \tan x$. $f(-x) = -f(x)$
Two formulas for the area of a triangle	$A = \frac{1}{2}bh$ $A = \frac{1}{2}ab\sin C$
Formula for the area of a circle	$A = \pi r^2$
Formula for the circumference of a circle	$C = 2\pi r$

Formula for the volume of a cylinder	$V = \pi r^2 h$
Formula for the volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Formula for the volume of a sphere	$V = \frac{4}{3}\pi r^3$
Formula for the surface area of a sphere	$A = 4\pi r^2$
Point-slope form of a linear equation	$y - y_1 = m(x - x_1)$

$\lim_{x \to 0} \frac{\sin x}{x}$	1
$\lim_{x \to 0} \frac{1 - \cos x}{x}$	0
Definition: A tangent line is…	…the line through a point on a curve with slope equal to the slope of the curve at that point.
Definition: A secant line is…	…the line connecting two points on a curve.
Definition: A normal line is…	…the line perpendicular to the tangent line at the point of tangency.

Definition: $f(x)$ is continuous at $x = c$ when…	1. $f(c)$ exists; 2. $\lim\limits_{x \to c} f(x)$ exists; and 3. $\lim\limits_{x \to c} f(x) = f(c)$
Limit definition of the derivative of $f(x)$: $f'(x) =$	$\lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
Alternate definition of derivative of f at $x = c$: $f'(c) =$	$\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$
What $f'(x)$ tells you about a function	• slope of curve at a point • slope of tangent line • instantaneous rate of change
Definition: Average rate of change is…	$\dfrac{\Delta y}{\Delta x} = \dfrac{f(b) - f(a)}{b - a}$

Power rule for derivatives: $\frac{d}{dx}\left(x^n\right) =$	nx^{n-1}
Product rule for derivatives: $\frac{d}{dx}\left(f(x)g(x)\right) =$	$f'(x)g(x) + g'(x)f(x)$ $= 1^{\text{st}} \cdot d(2^{\text{nd}}) + 2^{\text{nd}} \cdot d(1^{\text{st}})$
Quotient rule for derivatives: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =$	$\frac{g(x)f'(x) - f'(x)g(x)}{\left(g(x)\right)^2}$ $= \frac{\text{lo} \cdot \text{d hi} - \text{hi} \cdot \text{d lo}}{\text{lo} \cdot \text{lo}}$
Chain rule for derivatives: $\frac{d}{dx}\left(f\left(g\left(x\right)\right)\right)$	$f'\left(g\left(x\right)\right) \cdot g'\left(x\right)$, or derivative of the outside function times derivative of the inside function
$\frac{d}{dx}(\sin x) =$	$\cos x$

$\frac{d}{dx}(\cos x) =$	$-\sin x$
$\frac{d}{dx}(\tan x) =$	$\sec^2 x$
$\frac{d}{dx}(\cot x) =$	$-\csc^2 x$
$\frac{d}{dx}(\sec x) =$	$\sec x \tan x$
$\frac{d}{dx}(\csc x) =$	$-\csc x \cot x$

$\frac{d}{dx}(\arcsin x) =$	$\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arccos x) =$	$-\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\arctan x) =$	$\frac{1}{1+x^2}$
$\frac{d}{dx}(\operatorname{arccot} x) =$	$-\frac{1}{1+x^2}$
$\frac{d}{dx}(\operatorname{arcsec} x) =$	$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

$\frac{d}{dx}(\operatorname{arccsc} x) =$	$-\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$
Derivative of natural log: $\frac{d}{dx}(\ln x) =$	$\frac{1}{x}$
Derivative of log base a: $\frac{d}{dx}(\log_a x) =$	$\frac{1}{x\ln a}$
Derivative of natural exponential function: $\frac{d}{dx}(e^x) =$	e^x
Derivative of exponential function of any base: $\frac{d}{dx}(a^x) =$	$a^x \ln a$

Derivative of an inverse function: $\frac{d}{dx}\left(f^{-1}(x)\right) =$	$\frac{1}{f'\left(f^{-1}(x)\right)}$; The derivatives of inverse functions are reciprocals.
Rolle's Theorem: If f is continuous on $[a, b]$, differentiable on (a, b), **and**…	… $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.
Mean Value Theorem for Derivatives: If f is continuous on $[a, b]$ and differentiable on (a, b), then…	there exists a value of $c \in (a, b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.
Extreme Value Theorem: If f is continuous on a closed interval, then…	… f must have both an absolute maximum and an absolute minimum on the interval.
Intermediate Value Theorem: If f is continuous on $[a, b]$, then…	… f must take on every y-value between $f(a)$ and $f(b)$.

If a function is differentiable at a point, then…	… it must be continuous at that point. (Differentiability implies continuity.)
Four ways in which a function can fail to be differentiable at a point	• Discontinuity • Corner • Cusp • Vertical tangent line
Definition: A critical number (a/k/a critical point or critical value) of $f(x)$ is…	…a value of x in the domain of f at which **either** $f'(x) = 0$ or $f'(x)$ does not exist.
If $f'(x) > 0$, then…	… $f(x)$ is increasing.
If $f'(x) < 0$, then…	… $f(x)$ is decreasing.

If $f'(x) = 0$, then…	… $f(x)$ has a horizontal tangent.
Definition: $f(x)$ is concave up when…	$f'(x)$ is increasing.
Definition: $f(x)$ is concave down when…	$f'(x)$ is decreasing.
$f''(x) > 0$ means that $f(x)$ is…	concave up (like a cup)
$f''(x) < 0$ means that $f(x)$ is…	concave down (like a frown)

Definition: A point of inflection is a point on the curve where…	… concavity changes.
To find a point of inflection,…	… look for where f'' changes signs, or, equivalently, where f' changes direction.
To find extreme values of a function, look for where…	… f' is zero or undefined (critical numbers).
At a maximum, the value of the derivative…	… f' changes from positive to negative. (First Derivative Test)
At a minimum, the value of the derivative…	… f' changes from negative to positive. (First Derivative Test)

The Second Derivative Test: If $f'(x) = 0$ and…	…$f''(x) < 0$, then f has a maximum; if $f''(x) > 0$, then f has a minimum.
Position function $s(t) =$	$\int v(t)\,dt$, the antiderivative of velocity
Velocity function $v(t) =$	$s'(t)$, the derivative of position, as well as $\int a(t)\,dt$, antiderivative of acceleration
Acceleration function $a(t) =$	$v'(t)$, the derivative of velocity, as well as $s''(t)$, the second derivative of position
A particle is moving to the left when…	…$v(t) < 0$.

A particle is moving to the right when…	…$v(t) > 0$.
A particle is not moving (at rest) when…	…$v(t) = 0$.
A particle changes direction when…	…$v(t)$ changes signs.
To find displacement of a particle with velocity $v(t)$ from $t = a$ to $t = b$, calculate this:	$\int_a^b v(t)\,dt$
To find total distance traveled by a particle with velocity $v(t)$ from $t = a$ to $t = b$, calculate this:	$\int_a^b \lvert v(t)\rvert\,dt$

Area between curves	$\int_{left}^{right} (top - bottom)\,dx$
Volume of a solid with cross-sections of a specified shape	$\int_a^b \begin{pmatrix} \text{area of} \\ \text{cross-section} \end{pmatrix} dx$
Volume using discs	$\int_a^b \pi r^2 dx$ "perpendiscular"
Volume using washers (discs with holes)	$\int_a^b \left(\pi R^2 - \pi r^2\right) dx$
Volume using shells	$\int_a^b 2\pi rh\,dx$ "parashell"

Area of a trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Trapezoidal rule for approximating $\int_a^b f(x)dx$	$\approx \frac{1}{2}h(y_0 + 2y_1 + 2y_2 + \cdots$ $+2y_{n-1} + y_n)$
Average value of $f(x)$ on $[a, b]$	$\frac{1}{b-a}\int_a^b f(x)dx$
Power rule for antiderivatives: $\int x^n dx =$	$\frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$)
Constant multiple rule for antiderivatives: $\int kf(x)dx =$	$k\int f(x)dx$ (A constant coefficient can be brought outside.)

$\int \sin x \, dx =$	$-\cos x + C$
$\int \cos x \, dx =$	$\sin x + C$
$\int \sec^2 x \, dx =$	$\tan x + C$
$\int \csc^2 x \, dx =$	$-\cot x + C$
$\int \sec x \tan x \, dx =$	$\sec x + C$

$\int \csc x \cot x \, dx =$	$-\csc x + C$
$\int \frac{dx}{x} = \int \frac{1}{x} dx =$	$\ln\lvert x\rvert + C$
$\int e^x dx =$	$e^x + C$
$\int \frac{dx}{\sqrt{1-x^2}} =$	$\arcsin x + C$
$\int \frac{dx}{1+x^2} =$	$\arctan x + C$

L'Hôpital's rule for indeterminate limits If $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$,	then $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$ if the new limit exists.
Mean Value Theorem for Integration: If $f(x)$ is continuous on $[a, b]$, then…	…there exists a $c \in [a, b]$ such that $f(c) = \dfrac{1}{b-a}\int_a^b f(x)\,dx$
Fundamental Theorem of Calculus (part 1) $\dfrac{d}{dx}\int_a^x f(t)\,dt =$	$f(x)$
Fundamental Theorem of Calculus (part 2) $\int_a^b f(x)\,dx =$	$F(b) - F(a)$, where F is an antiderivative of f
A differential equation is…	…an equation containing one or more derivatives.

To solve a differential equation,...	...first separate the variables (if needed) by multiplying or dividing, then integrate both sides.
Exponential Growth and Decay: If $\frac{dy}{dt} = ky$, then…	$y = Ce^{kt}$, where C is the quantity at $t = 0$, and k is the constant of proportionality.
$\int_a^b (\text{rate of change})\, dt =$	the amount which that quantity has changed from $t = a$ to $t = b$